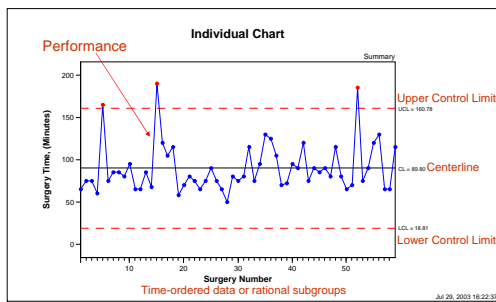


## Control Chart Theory

## Construction of Control Charts

- Common features
  - Centerline based on process average
  - Control limits based on sigma or t-limits

## Construction of Control Charts



## Construction of Control Charts

- Differences
  - Distributions determining the calculation of control limits and centerline and application of tests for special cause
    - Gaussian (normal) distribution
    - Binomial distribution
    - Poisson distribution
    - Geometric distribution
  - Time-ordered or rationally sub-grouped data
  - Constant or variable sub-group size – straight or step-pattern control limits

## Apply to All Control Charts

- Distinguish between *common-cause* variation and *special-cause* variation
- For the usual case (*no standard given*) the control limits are calculated from the data
- Control limits are traditionally set at three standard deviations (3-sigma or  $3\sigma$ ) limits, which would be expected to include 99.73% of the plotted points if the data were normally distributed

## For Practical Purposes

- Even when the data is not perfectly normally distributed, 3-sigma levels have been found to give good practical results

## Motto

- “This process is in control” really means:
  - For practical purposes, it pays to act as if no special-cause variation were present.

## Statistical Process Control

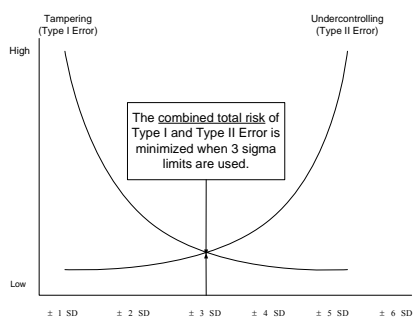
- Type I errors
  - When people make a Type I error, they conclude there is a problem when, in fact, there is not, and
  - they “tamper” with a process that is already stable

## Statistical Process Control

- Type II errors
  - When people make a Type II error, they conclude that there is no problem when, in fact, a special-cause actually exists, and
  - they fail to intervene in the process when they should

## Control Chart – Type of Error

Based on Control Chart	Process	
	Common cause variation present	Special cause variation present
Do Not Take Action	Correct	Type II Error
Take Action	Type I Error	Correct



## Trial by Jury

Verdict of Jury	Defendant	
	Innocent	Guilty
Not Guilty	Correct	Incorrect
Guilty	Incorrect	Correct

## Hypothesis Testing

Result of Test	Population	
	$\mu = \mu_0$	$\mu \neq \mu_0$
Do Not Reject	Correct	Type II Error
Reject	Type I Error	Correct

## Probability of Making an Error

Based on Control Chart	Process	
	Common cause variation present	Special cause variation present
Do Not Take Action	Correct	$\beta$ -risk
Take Action	$\alpha$ -risk	Correct (Power=1- $\beta$ )

## $\alpha$ -Risk

- Alpha ( $\alpha$ ) risk is defined as the risk or probability of rejecting the null hypothesis when, in fact, it is true.
  - Synonymous with Type I error and tampering with the process
  - In other words, stating that a difference exists when actually there is none. Alpha risk is stated in terms of probability (such as 0.05 or 5%).
- The value (1-alpha) corresponds to the confidence level of a statistical test, so a level of significance alpha = 0.05 corresponds to a 95% confidence level

## $\beta$ -Risk

- Beta ( $\beta$ ) risk is defined as the risk or probability of accepting the null hypothesis when, in fact, the alternate hypothesis is true.
  - In other words, stating that no difference exists when actually there is
- It is the probability of under-controlling, not taking action when action is actually warranted

## The Power

- The value (1-beta) is known as the "power" of a statistical test. The power is defined as the probability of rejecting the null hypothesis, given that the null hypothesis is indeed false
- In terms of control charts, it is the probability of correctly identifying special cause variation

### Number of Required Data Points

- Ten points may be sufficient for process improvement if their results suggest non-random influence, but *not* sufficient if the results suggest a state of control
- Usually, in order to infer safely that a state of control exist, at least 24 consecutive data points are required

### Conclusion

- Control charts are useful tools for performance improvement even with few data points
- However, in order to conclude with confidence that a process does *not* experience special-cause variation, more data points (usually 24) are needed

### “T-Sigma” versus “3-Sigma” Limits

- With about 25 data points, conventional 3-sigma limits are most effective for
  - providing a good balance between offering sufficient power for the detection of special-cause variation
  - for maintaining a satisfactory total  $\alpha$ -risk of getting one more points outside the control limits by random chance

### “T-Sigma” versus “3-Sigma” Limits

- However, 3-sigma limits often do not maintain a satisfactory  $\alpha$ -risk when the number of data points differs greatly from 25

### “T-Limits”

- Have been developed to keep the  $\alpha$ -risk to less than 0.09 for fewer data points
- They should be used in particular for small numbers of data points
- They apply both to control charts for attribute and variables data

### T-Limits

# of plotted points	T
2	1.5
3 – 4	2.0
5 – 9	2.5
10 – 34	3.0
35 – 199	3.5
200 – 1500	4.0

Exception: for “standard given” use T=2 for 1 or 2 data points

## Critical Processes

- For process improvement 3-sigma limits are usually used (and not 3.5 or 4.0) even with more data points
- For critical processes, particularly for adverse outcomes (major surgical complications), 2-sigma limits may sometimes be appropriate
- The increased  $\alpha$ -risk is balanced against the better chance of detecting problems early